Méthodes topologiques en analyse non linéaire:développements récents -
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Symplectic reduction for differential equations degenerating on the boundary

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We study the geometry underlying semiclassical asymptotic solutions for a class of (pseudo)differential equations degenerating on the boundary ∂X of a smooth compact manifold X. We represent X as the quotient of a closed smooth manifold M by a semi-free circle action τ, ∂X being an isomorphic projection of the fixed-point set of τ, and define the phase space Φ as the symplectic reduction of T^*M with respect to the circle action $\hat\tau$ induced by τ. Although the manifold Φ (equipped with the natural projection onto X) is isomorphic to T^*X over the interior of X, it fails to be so in a neighborhood of the boundary (where it is even not a locally trivial bundle). This prevents one from using the standard Maslov canonical operator for constructing asymptotic solutions u associated with Lagrangian submanifolds $\Lambda\subset\Phi$. However, Λ can be viewed as the quotient of a $\hat\tau$-invariant Lagrangian manifold $L\subset T^*M$ and u as a τ-equivariant asymptotic solution of an appropriate lift of the original equation to M. Accordingly, u can be written via Maslov's canonical operator on L, and it remains to represent the resulting expression via Φ, Λ, and other objects related to X alone.